

**MATHEMATICS  
SPECIALIST  
UNITS 3 AND 4**

**Section One:  
Calculator-free**

If required by your examination administrator, please place your student identification label in this box

Student number: In figures

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In words

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Your name

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**Time allowed for this section**

Reading time before commencing work: five minutes

Working time: fifty minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	97	65
				<b>Total</b>	100

## Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free**

**35% (53 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

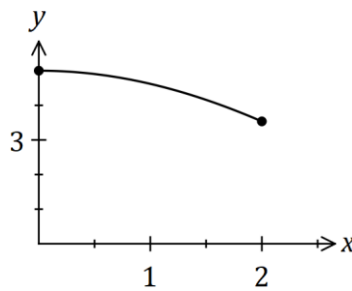
Working time: 50 minutes.

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**Question 1**

**(4 marks)**

The curve defined by  $y = 5 \cos\left(\frac{\pi x}{8}\right)$ , where  $0 \leq x \leq 2$ , is shown below.

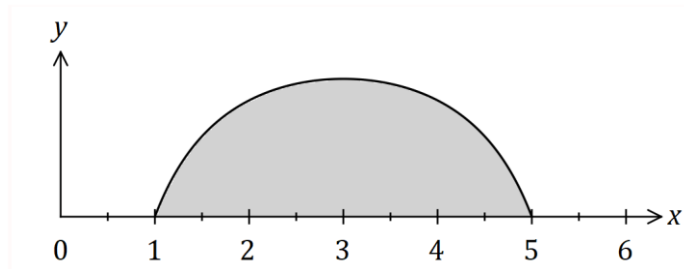


Determine the volume of the solid generated when the area bounded by the  $x$  axis and the curve is rotated  $360^\circ$  about the  $x$  axis between  $x = 0$  and  $x = 2$ .

Question 2

(5 marks)

Part of the graph of  $y = 1 + \frac{5}{x(x-6)}$  is shown below.



Determine the shaded area, bounded by the curve and the  $x$ -axis.

**Question 3**

**(4 marks)**

Consider the equation  $9z^3 - 18z^2 + 5z - 10 = 0$ .

(a) Show that  $z = 2$  is a solution of the equation.

(1 mark)

(b) Determine 2 other solutions of the equation.

(3 marks)

Question 4

(9 marks)

$$\text{Let } w = \frac{-1 + i}{-1 + \sqrt{3}i}.$$

(a) Determine the real constants  $a$  and  $b$ , where  $w = a + ib$ . (2 marks)

(b) By first expressing  $-1 + i$  and  $-1 + \sqrt{3}i$  in polar form, write  $w$  in polar form. (3 marks)

(c) Hence determine an exact value for  $\sin\left(\frac{\pi}{12}\right)$ . (2 marks)

(d) Determine  $w^6$  in Cartesian form. (2 marks)

**Question 5**

**(8 marks)**

Two planes have equations  $x - y + 2z - 10 = 0$  and  $2x - y + z - 9 = 0$ .

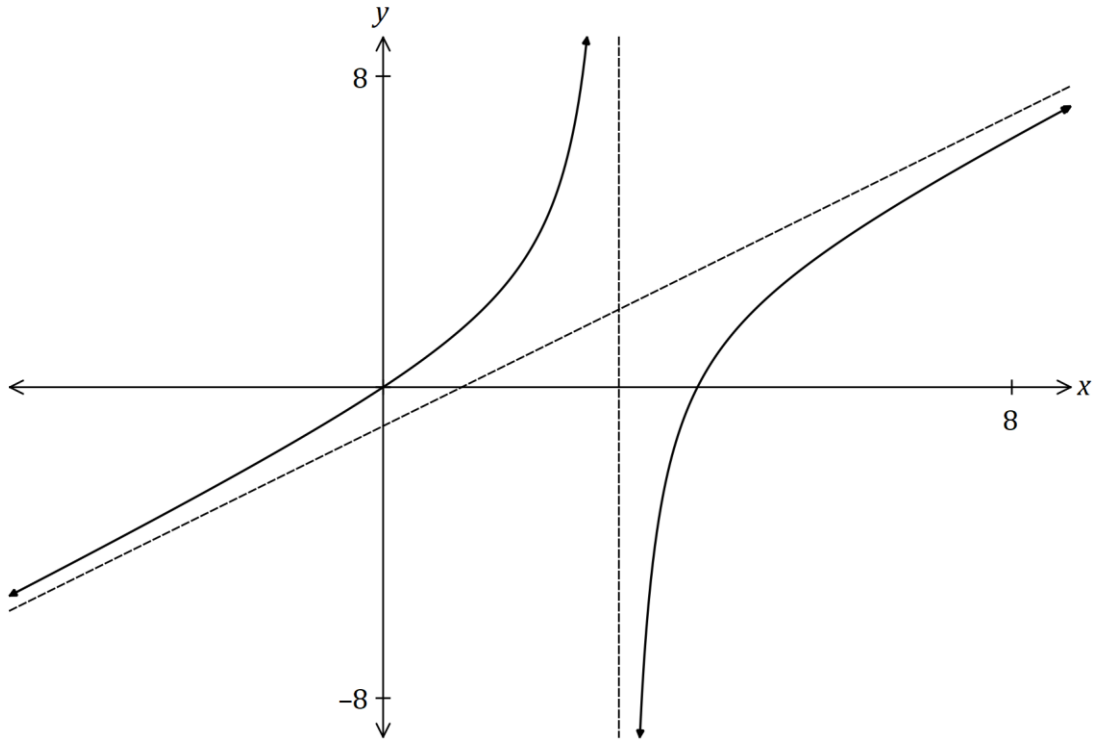
- (a) Determine the Cartesian equation of a third plane that is perpendicular to these planes and passes through the point  $(2, -2, 3)$ . (4 marks)

- (b) Determine the point of intersection of all three planes. (4 marks)

Question 6

(9 marks)

The graph of  $y = \frac{x^2 - 4x}{x - 3}$  and its two asymptotes is shown below.

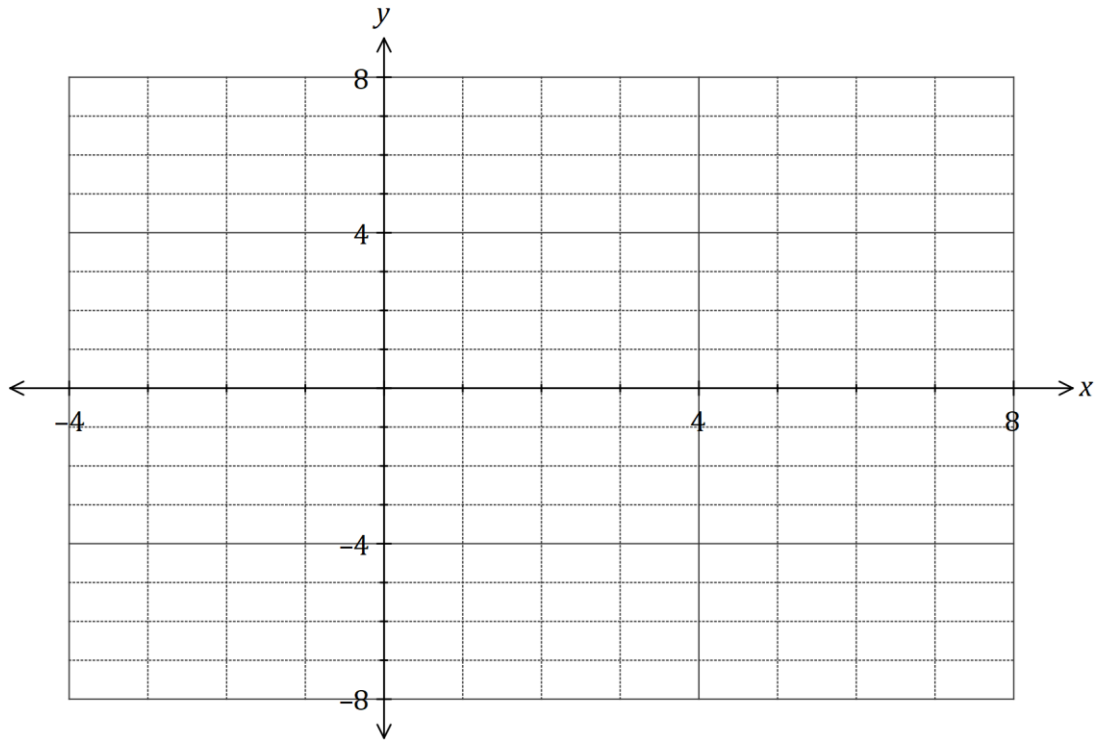


(a) Determine the equation of both asymptotes.

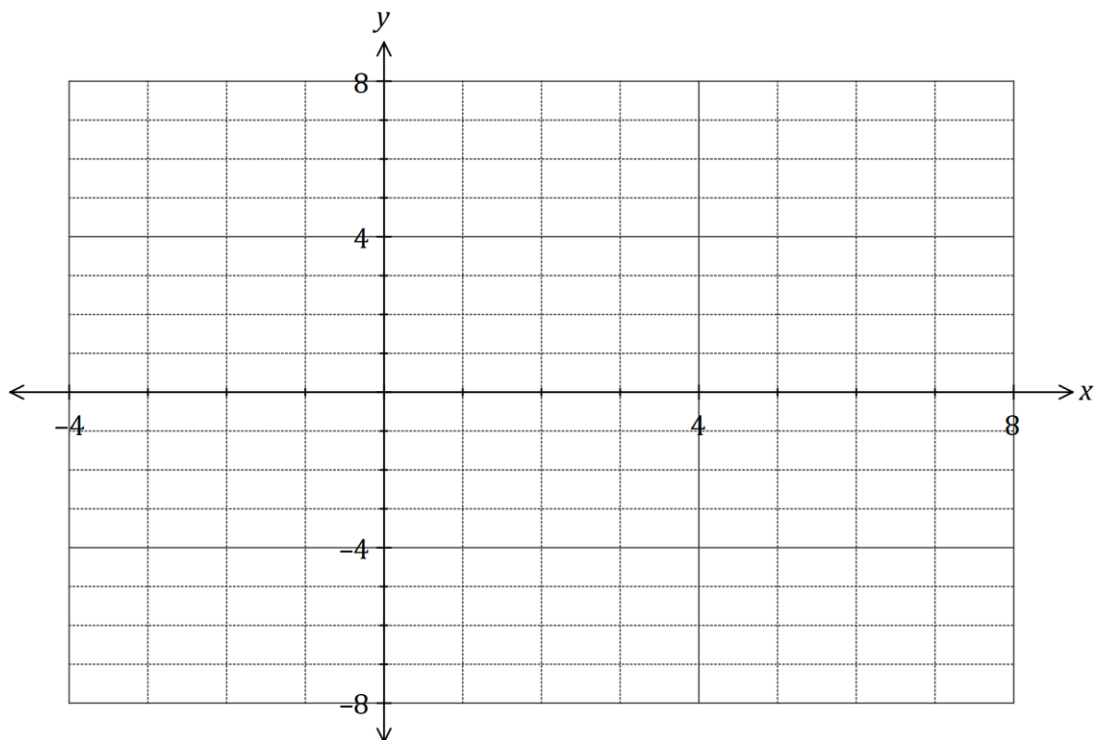
(3 marks)



- (b) On the axes below, sketch the graph of  $y = \frac{x - 3}{x^2 - 4x}$ . (4 marks)



- (c) On the axes below, sketch the graph of  $y = \frac{x^2 - 4|x|}{|x| - 3}$ . (2 marks)



Question 7

(7 marks)

Function  $f$  is defined as  $f(x) = \sqrt{1 - 2x}$  and function  $g$  is defined as  $g(x) = \ln(5 + x)$ .

(a) Determine a rule for  $f^{-1}(x)$ , the inverse of  $f$ , and state its domain and range. (3 marks)

(b) Determine an expression for  $f \circ g(x)$  and state its domain. (4 marks)

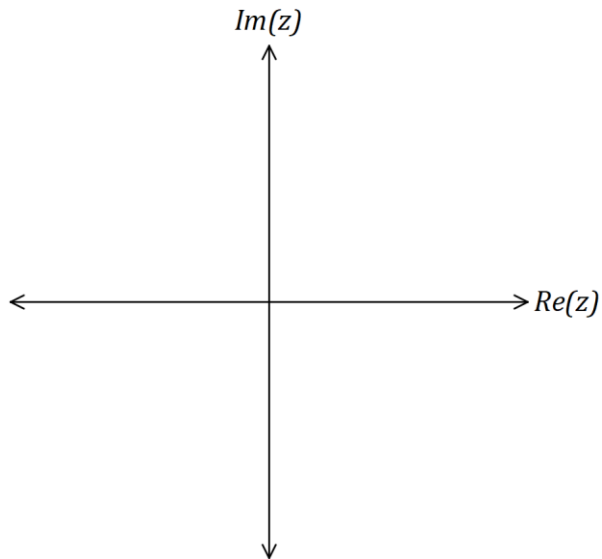
Question 8

(7 marks)

On the Argand planes below, sketch the locus of the complex number  $z$  given by the following.

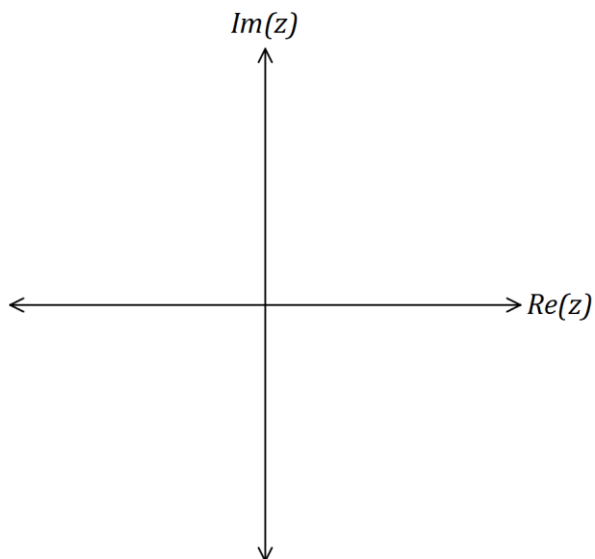
(a)  $|z - 1 - i| = |z + 1 - 3i|$ .

(3 marks)



(b)  $\frac{1}{z} + \frac{1}{\bar{z}} = \frac{1}{3}$ ,  $z \neq 0$ .

(4 marks)



End of questions

Supplementary page

Question number: \_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_

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